A Note on Tractability and Artificial Intelligence

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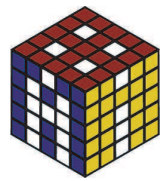
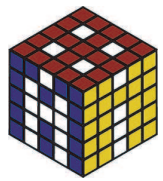


Computer Metaphor and Church-Turing Thesis

Famous ideas at the heart of many endeavors in computational cognitive modeling AI:

1. “Computer metaphor” of the mind (i.e. the concept of a computational theory of mind).
2. Church-Turing thesis.

1. bridges gap between humans and computers:
   - Human mind and brain can be seen as information processing system.
   - Reasoning and thinking corresponds to computation as formal symbol manipulation.
2. gives account of the nature and limitations of the computational power of such a system.



P-Cognition Thesis

Significant impact on cognitive science and cognitive psychology:

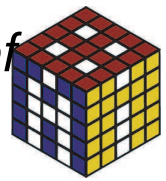
- Explain human cognitive capacities modeled in terms of computational-level theories (i.e., as precise characterizations of hypothesized inputs and outputs of respective capacities together with functional mappings between them).
- Problem: Computational-level theories often underconstrained by available empirical data!

$\Rightarrow$ Use mathematical complexity theory as assisting tool:
Concept of **NP-completeness**!

P-Cognition thesis

Human cognitive capacities are hypothesized to be of the polynomial-time computable type.

(*Interpretation: "Humans can comfortably solve non-trivial instances of this problem, where the exact size depends on the problem at hand".*)



“polynomial-time computable” = “efficient”?

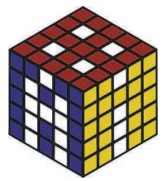
- Humans able to solve problems which may be hard in general but feasible if certain parameters of the problem restricted.
- **Parametrized complexity theory**: “tractability” captured by FPT.[1]

FPT-Cognition thesis

Human cognitive capacities to be fixed-parameter tractable for one or more input parameters that are small in practice (i.e., computational-level theories have to be in FPT).

$\Rightarrow$ (Re)introducing idea of tractable computability for cognition into AI and cognitive systems research by rephrasing FPT-form of Tractable Cognition thesis into “**Tractable AGI thesis**” (Tractable Artificial and General Intelligence thesis).

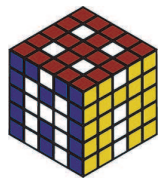
[1] A problem P is in FPT if P admits an $O(f(\kappa)n^c)$ algorithm, where n is the input size, κ is a parameter of the input constrained to be “small”, c is an independent constant, and f is some computable function.



The Tractable AGI Thesis

- All of currently available computing systems are ultimately finite systems with limited resources.
- In close analogy to FPT-Cognition thesis demand for computer models of AI and complex cognitive capacities to be of the (at least) fixed-parameter tractable type!

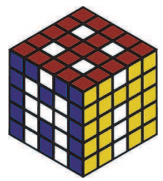
Tractable AGI thesis
Models of cognitive capacities in artificial intelligence and computational cognitive systems have to be fixed-parameter tractable for one or more input parameters that are small in practice (i.e., have to be in FPT).



Worked Example (1)

- Classical attempts at modeling and reproducing human rationality and reasoning in artificial systems based on logical formalisms.
- Having to deal with the defeasibility of common sense reasoning: Development of non-monotonic logics.
- Severe drawback: All prominent approaches to logic-based non-monotonic reasoning shown to be highly intractable (i.e., NP-hard or worse).
- Assuming P $\neq$ NP: No efficient algorithm exactly computing processes of reasoning exactly for any set of input parameters.

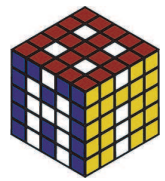
$\Rightarrow$ Progressing abandonment of the logical approach to modeling cognitive capacities and processes within cognitive psychology and later also AI.
$\Rightarrow$ New Approach: Bayesian probabilistic modeling.



Worked Example (2)

- Unconditional optimism not justified: Switch of formalisms has not made intractability issue disappear!
- Probabilistic (Bayesian) inference of most probable explanation (MPE) of set of hypothesis given observed phenomena: NP-hard!
- Worse: Stays computationally intractable under approximation.
- Entire abandonment of overall approach most likely premature: Different proposals for restoring tractability, e.g., proposing that restricted input domains might be used in human cognition.
- **Tractable AGI thesis**: Principle for deciding whether a particular Bayesian-style model is worth investing further effort!
- Suppose model at hand shown to be fixed-parameter tractable for some set of parameters κ:
  Parameters in $\kappa =$ fixed small constants for problem instances realized in practice. $\Rightarrow$ Possible to efficiently compute a solution!

$\Rightarrow$ Non-trivial corollary: Any instance of problem can be reduced to **problem kernel**.



Kernelization and Kernelizability (1)

Kernelization

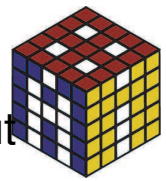
Let P be a parameterized problem. A kernelization of P is an algorithm which takes an instance x of P with parameter κ and maps it in polynomial time to an instance y such that $x \in P$ if and only if $y \in P$, and the size of y is bounded by $f(\kappa)$ (f a computable function).

Kernelizability

A problem P is in FPT if and only if it is kernelizable.

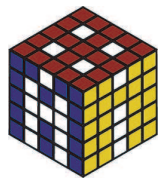
1. Any positive FPT result implies existence of "downward reduction" for underlying problem to some sort of smaller or less-complex instance of same problem.
2. Assuming W[1] $\neq$ FPT:[2] Any negative result implies that there is no such downward reduction.

[2] W[1] is the class of problems solvable by constant depth combinatorial circuits with at most 1 gate with unbounded fan-in on any path from an input gate to an output gate. W[1] $\neq$ FPT can be seen as analogous to P $\neq$ NP.



Kernelization and Kernelizability (2)

- (Formal) correspondence between complex instances and simpler manifestations of problem matches well with observation from problem-solving and reasoning experiments with human participants: Different forms of reductions from complex to simpler (but still solution-equivalent) problems reported to be pervasive and crucial.
- Described equivalence forms connecting point to developments in cognitive science/cognitive psychology (*heuristics program*):
  - Humans in common sense reasoning do not apply full-fledged form of logical or probabilistic reasoning to highly complex problems, rely on mechanisms reducing problems to equivalent, simpler ones.
- Kernelization/Kernelizability can provide inspiration and first hints at hypothesizing specialized cognitive structure capable of computing reduced instance of problem.

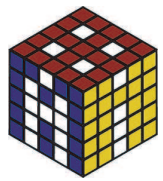


To be continued!

If you are interested in tractability and similar topics in A(G)I (and/or cognitive science), you are happily invited to...

1. ...talk to me after the session.
2. ...contact us by mail:

   tbesold@uos.de or **robere@cs.toronto.edu**.
3. ...attend the talk on "**When Thinking Never Comes to a Halt: Tractability, Kernelization and Approximability in AI**" at *Philosophy and Theory of AI (PT-AI) 2013* at Oxford (UK) on September 21 & 22, 2013. (Or have a look at the proceedings...)



A look ahead... (1)

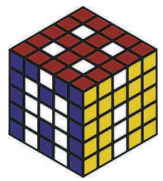
Complementary perspective on heuristics:

- Approximation theory!
- Instead of precisely solving kernel, compute approximate solution to original problem (i.e., solution to relaxed problem).

Constant-Factor Approximation

An optimization problem P is in APX if P admits a constant-factor approximation algorithm, i.e., there is a constant factor $\varepsilon > 0$ and an algorithm which takes an instance of P of size n and, in time polynomial in n, produces a solution that is within a factor $1 + \varepsilon$ of being optimal (or $1 - \varepsilon$ for maximization problems).

- If constant bounding approximation ratio meaningfully chosen w.r.t. problem at hand, constant-factor approximation allows for natural modeling of Simon's "satisficing".



A look ahead... (2)

Fixed-Parameter Approximability

The fixed-parameter version P of a minimization problem is in FPA if there is an algorithm such that for any given problem instance I with parameter k, and question $OPT(I) \leq k$, the algorithm which runs in $O(f(k)n^c)$ (where $n = |I|$) either outputs "no" or produces a solution of cost at most $g(k)$ (where f and g are recursive functions, and g is linear in the constant k).

Fixed-Parameter Approximable AGI thesis

Models of cognitive capacities in artificial intelligence and computational cognitive systems have to be fixed-parameter approximable for one or more input parameters that are small in practice (i.e., have to be in FPA).

- APX and FPT with witness independently imply FPA.
- Interpretation level: FPA combines both views of heuristics!
  1. Approximability character accommodating for notion of satisficing.
  2. Fixed-parameter character accounting for possibility of complexity reduction by keeping key parameters of problem fixed.