Bounded Kolmogorov Complexity Based on Cognitive Models

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Pattern discovery

- In nature, discovering spatial and temporal patterns can be critical for finding resources and avoiding dangers
  - Strawberries ripen in June
  - Chanterelles grow on north slopes
  - Where there are seagulls there is fish
  - That bear is aggressive
  - There will be a storm tonight.

Relevance to AGI

- Evolutionary selection pressure on the ability to discover patterns.
- Strong correlation in humans between pattern discovery and general intelligence.
- AGI agents should be able to do pattern discovery on the human level and beyond!

IQ tests

- Pattern discovery problems dominate modern IQ tests, particularly
  - Progressive matrix problems
  - Number sequence problems

Number sequence problems

105,107,109,?

Number sequence problems

105,107,109,111

Number sequence problems

2,2,4,8,32,?

Number sequence problems

2,2,4,8,32,256

How did you solve these problems?

- Did you go
  - from 105 to 107
    - By adding 2
  - from 2 and 2 to 4
    - By adding them or multiplying them
  - from 4 and 8 to 32
    - By multiplying them

Did you look for short expressions that describe these transitions?

Ockhams's razor

- Kolmogorov complexity
  - Uncomputable
- Solomonoff probability/complexity
  - Uncomputable
- Levin complexity
  - Computable
  - Depends on two arbitrary choices
    - The universal TM U
    - The log of the runtime

State-of-the-art tools

- Tools for solving number sequence problems
  - Mathematica
  - Maple
  - Online Encyclopedia of Integer Sequences
  - Wolfram-Alpha

- These tools all performed below average human level in our evaluation (shown later).

- So something else seems to be needed.

Subjectivity

1,2,?

Argument	Solution
Repetition	1
Mirroring	2
Successor	3
Double	4

Subjectivity: our answers depend

- On what we prefer
  - 1,2,?
- On what we know
  - 0,1,4,6,13,?
- On what we can compute
  - 2,2,4,8,32,256,?

Subjectivity

- Wittgenstein: We can always argue for any continuation of any number sequence.

- But in real life our predictions are either correct or incorrect. We get rewarded if we are right and punished if we are wrong!

- The same holds for IQ tests…

Taking the consequences

- Taking subjectivity into account, we will relativize our computational model to a subject, i.e. a cognitive model.

- We use a rough cognitive model of a (strong) human problem solver.
  - A notion of term
  - A framework for evaluating terms.

Cognitive model

- Vocabulary
  - Digits: 0,1,…,9
  - Operators: +,-,*,/ and digit juxtaposition
  - Unary function symbol: f
  - Variable: n
- Terms
  - as expected

Cognitive model

- Sequences are generated by such terms.
  - 103,105 is generated by f(n-1)+2
  - 2,2,4,8,32 is generated by f(n-1)*f(n-2)
  - 1,2,6,24,120,… is generated by f(n-1)*n

Cognitive model

- A framework for evaluating terms.
  - Standard table entries
    - 5*5=25
    - 87+6=13
  - Standard algebraic rules
    - x+y=y+x
    - x+(y+z)=(x+y)+z
  - A maximum term size (working memory) of 8
  - An algorithm for evaluating terms with these rules.

Our complexity measure

- The length of the shortest term that generates the input sequence with the following additional restriction:

- All positions of the input sequence plus the next position must be computable in the cognitive model.

Performance on IQ tests

Program	PJP	PA	IST
SeqSolver	11	29	28
Mathematica	6	9	9
Maple	6	6	9
OEIS	4	11	11
WolframAlpha	6	9	12

Generality

- Many pattern discovery problems can be encoded as number sequence problems.

- We used the language of arithmetic in this study, but the same strategy works for other terms and term rewriting systems as well.

Conclusion

- In AGI we aim for human level performance (and beyond). To that end we may exploit the strengths, but also the limitations of human reasoning.

- We exploited limitations in the human cognitive resources in order to reduce the size of the search space (and thereby the computational complexity).

- We suggested a version of Kolmogorov complexity that is
  - feasibly computable
  - derived from the goal of reaching human level performance
  - applicable to arbitrary term-rewriting systems.

- Encouraging results on IQ test problems, but more work is needed.